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Sharpening Enhancement of Digitized Mammograms with Complex Symmetric Daubechies Wavelets*

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Abstract–**Some complex symmetric Daubechies wavelets provide a natural way to calculate zero-crossings because of a hidden "Laplacian operator" in the imaginary part of the scaling function. We propose a simple multiscale sharpening enhancement algorithm based on this property. The algorithm is tested on low-contrast digitized mammograms.**

Keywords: **Image processing, wavelets, mammograms.**

I. INTRODUCTION

Many breast cancers cannot be detected on the basis of mammographic images. This is partly due to the fact that mammograms are poor quality images with low-contrast resulting from the small differences in X-ray attenuation between breast tissues. A very actual challenge is to improve the visual quality of mammograms by numerical processing in order to help in the early detection of cancer.

The standard method to improve the local contrast of an image is to subtract the Laplacian of the signal from the original signal. Some very promising wavelet-based algorithms have been proposed that extend this method to allow multiscale processing using wavelet representations [1,2]. The wavelets used in these studies are of various kinds: discrete separable, discrete non-separable (hexagonal) or continuous. The aim of the present paper is to show that complex Symmetric Daubechies Wavelets (SDW) have also a great potential in such processing [3].

II. SYMMETRIC DAUBECHIES WAVELETS

We recall that a Daubechies wavelet $\psi(x)$ is constructed from a (in general complex) scaling function $\varphi(x) = h(x) + ig(x)$, through the multiresolution analysis procedure [4]. In the construction, three constraints are imposed on the function $\varphi(x)$: compact support inside the interval $[-J, J+1]$ (for some non-negative integer J), orthogonality of the discrete translates and regularity. The SDW are obtained by imposing an additional symmetry constraint on $\varphi(x)$. This constraint restricts J to even numbers and implies that $\varphi(x)$ and $\psi(x)$ are even and odd functions respectively (with respect to $x = 1/2$) [3].

SDW have many interesting properties for numerical simulations and image processing [5-7]. One of them is

$$g(x) \cong \alpha d^2 h(x)/dx^2 \tag{1}$$

where α is a real parameter (for example, α=0.154 for $J = 2$ and α=0.102 for $J = 4$). Thus, the imaginary part of $\varphi(x)$ inherits a Laplacian operator from the symmetry constraint. It has been shown that the approximation (1) is very accurate for the first values of J and on most of the frequency range $[0, \pi]$, where π is the normalized Nyquist frequency (sampling steps are normalized to unity) [6].

III. A SHARPENING ENHANCEMENT ALGORITHM

According to the multiresolution analysis with tensor product bases, an image $I(m, n)$ is projected onto some "approximation" space generated by the dyadic translations of the scaled function $\varphi(x)\varphi(y)$ (at the resolution scale j_{max} of the original image). If we denote the complex projection coefficients by $c_{m,n}^{j_{max}} = h_{m,n}^{j_{max}} + ig_{m,n}^{j_{max}}$, then we can estimate $h_{m,n}^{j_{max}}$ and $g_{m,n}^{j_{max}}$ with the following three steps iterative procedure. 1) Start from the usual approximation

$$h_{m,n}^{j_{max}} = I(m, n) \tag{2}$$

2) Evaluate $h_{m,n}^{j_{max}+1}$ using a one-level synthesis operation with the real part of the inverse SDW kernel only. 3) Make a one-level complex wavelet transform. The result is a quite accurate estimation of the real and imaginary parts of the projection coefficient $c_{m,n}^{j_{max}}$. In a first approximation, $h_{m,n}^{j_{max}} \cong I(m, n)$ and $g_{m,n}^{j_{max}}$ is proportional to the Laplacian of the $I(m, n)$.

A N-level wavelet transform W can be represented as

$$\{c_{m,n}^{j_{max}}\} \xrightarrow{W} \{c_{m,n}^{j_{max}-N}, \mathbf{d}_{m,n}^{j_{max}-N}, \ldots \mathbf{d}_{m,n}^{j_{max}-1}\} \tag{3}$$

where the quantities $\mathbf{d}_{m,n}^{j_{max}-k}$ represent the set of coefficients for the three wavelet sectors. The complex scaling wavelet coefficients $c_{m,n}^{j_{max}-N}$ result from the nested actions

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of the complex low-pass filter. Many enhancement algorithms can be devised and implemented in the multiresolution space as a local transformation on the complex scaling and wavelet coefficients during the reconstruction of the signal. For instance, a multiresolution sharpening operation can be implemented through

$$\tilde{c}^k_{m,n} = c^k_{m,n} - \rho_k g^k_{m,n}/2^{2(j_{max}-k)} \tag{4}$$

where the coefficients $c^k_{m,n}$ are induced by the synthesis process, while the coefficients $g^k_{m,n}$ are calculated during the decomposition process.

Preliminary tests of this algorithm have been performed on low-resolution and low-contrast mammographic images. A typical result is shown on Figures 1 and 2. The original mammogram (Figure 1) has a resolution of 400 micron/pixel with pixel values ranging from 668 to 3356 and coded at 2 byte/pixel. We have processed it with the maximally regular SDW of order $J = 2$, over a 3-level multiresolution decomposition ($N = 3$) and with enhancement parameters $\rho = \{20, 5, 1\}$. The local contrast between the low and high X-ray attenuation regions has been improved significantly in the processed mammogram (Figure 2). For better visual comparison of the sharpening enhancement, the pixels of the processed mammogram with low and high values outside the original window range, have been clipped to 668 and 3356 respectively.

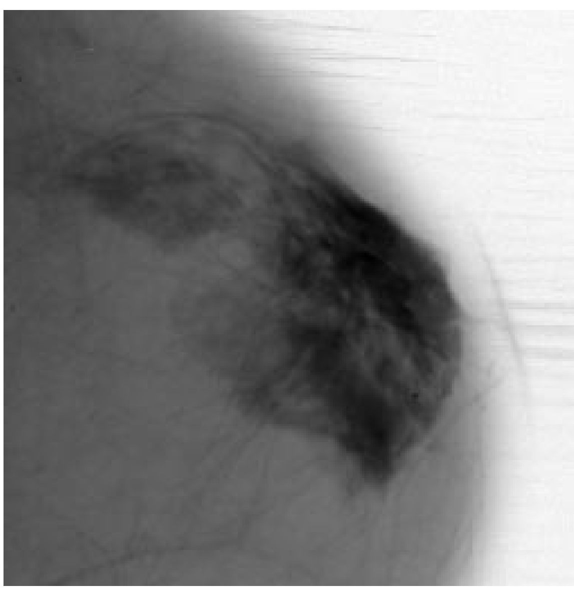

Figure 1: Original mammogram

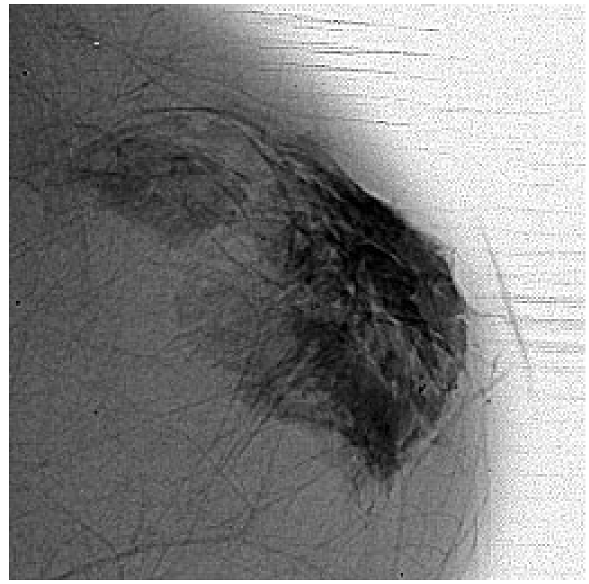

Figure 2: Processed mammogram with $N = 3$, $J = 2$ and $\rho = \{20, 5, 1\}$

IV. CONCLUSION

We have proposed a simple multiresolution sharpening enhancement operation based on the "hidden" zero-crossing property of the complex symmetric Daubechies wavelets. The algorithm is self-contained in the sense that all the necessary operations (interpolation and second derivative) are executed with the complex symmetric kernels. We can improve still more the visual quality by coupling the above procedure with a robust denoising algorithm

$$\tilde{\mathbf{d}}^k_{m,n} = \eta(\mathbf{d}^k_{m,n}, \mathbf{t}_k) \tag{5}$$

where η is a thresholding function and $\mathbf{t}_k$ a set of adaptive thresholds [8]. We are currently pursuing this development, as well as quantitative comparison with other contrast enhancement algorithms.

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