Synthesis by Language Equation Solving: Extended Abstract

Nina Yevtushenko[¶] Tiziano Villa[§] Robert K. Brayton[†]

Alex Petrenko[‡]

Alberto L. Sangiovanni-Vincentelli[†]

[¶] Dept. of EECS, Tomsk State University, Tomsk, 634050, Russia

[§] PARADES, Via di S.Pantaleo, 66, 00186 Roma, Italy

[†] Dept. of EECS, Univ. of California, Berkeley, CA 94720

[‡] CRIM, 1081 McGill College Av., Montreal, H3A2N4, Can

March 11, 2000

Abstract

The problem of designing a component that, when combined with a known part of a system called the context, conforms to a given overall specification arises in applications ranging from logic synthesis to the design of discrete controllers.

We cast this as solving abstract equations over languages and study their most general solutions. We specialize such language equations to various languages associated with important classes of automata used for modeling systems, e.g., regular languages as counterparts of finite automata, FSM languages as counterparts of FSMs. We operate algorithmically on these languages through their automata and study how to solve effectively their language equations. We study also the maximal subsets of solutions closed with respect to various language properties.

Finally we apply this theory to sequential logic synthesis and derive results for some commonly studied topologies of networks of FSMs. This paper is mostly a survey and unification of existing results, sometimes in disparate parts of the scientific literature, that attempts to put them into a common framework and notation.

An important step in the design of complex systems is the decomposition of the system into a number of separate components which interact (communicate) in some well-defined way. In this context, a typical question is how to design a component that combined with known parts of the system, called the context, conforms to a given overall specification. This question arises in several applications ranging from logic synthesis to the design of discrete controllers. We consider three key issues:

- how to model the system, its components, and the specification,
- how to model the composition of the components (i.e. the communication between the components), and
- how to model the notion of a system conforming to a modeled specification.

Regarding the first issue, different types of mathematical machines are used to model the components of a system: finite automata (FA), finite state machines (FSMs), Petri Nets (PNs), ω-automata (ω-FA) and games are the ones most commonly used. Once the first issue is decided, related choices must be made for the other two. For instance, if FSMs are used to model the system, operators for composing FSMs must be introduced together with a notion of an FSM conforming to another FSM. For the latter point, popular choices are language containment or simulation of one FSM by the other; regarding FSM composition, various forms have been described in the literature. Thus one can define an equation over FSMs of the type $M_A \odot M_X \preceq M_C$, where M_A models the context, M_C models the specification, M_X is unknown, $\odot$ stands for a composition operator and $\preceq$ for a conformance relation (e.g. $\subseteq$, language containment). Therefore for each model of mathematical machines and how they communicate, appropriate equations can be set up and their solutions investigated. Also more complex equations or systems of equations can be formulated depending on the topology of the system's interconnections.

A key observation is that a certain class of languages is associated with each model of a mathematical machine, and each method for their communication corresponds to operations on pair of languages. Therefore we define and investigate first,

1. languages
2. operations on them, and
3. abstract equations over them.

We discuss two composition operators for abstract languages:

1. synchronous composition, $\bullet$, and
2. parallel composition, $\diamond$,

and check conformity by language containment. Roughly, synchronous composition corresponds to instantaneous communication and parallel composition to machines communicating asynchronously, allowing for arbitrary finite delays between communication events.

We study the most general solutions of the language equations $A \bullet X \subseteq C$ and $A \diamond X \subseteq C$, determining what language operators are needed to express them. Then we specialize such equations to languages associated with particular classes of automata used for modeling systems, e.g., regular languages as counterparts of finite automata and FSM languages as counterparts of FSMs. At this point, we can operate algorithmically on those languages (since we have finite representations) through their automata and associated operations on them, and study how to solve effectively their language equations. A key point is to find all solutions that are within the same language class, e.g., when studying FSM language equations, we look for solutions that are FSM languages. Moreover, we may be interested to subsets of solutions characterized by some further property of practical interest, e.g., FSM languages that satisfy the Moore property, and so the valid solutions are restricted further. For each such restriction we give appropriate algorithms that operate on the underlying finite representation.

As an example, consider FSM equations under synchronous composition. Given alphabets I_1, I_2, U, V, O_1, O_2, an FSM M_A over inputs $I_1 \times V$ and outputs $U \times O_1$, an FSM M_C over inputs $I_1 \times I_2$ and outputs $O_1 \times O_2$, define the FSM equation

$$M_A \bullet M_X \subseteq M_C,$$

whose unknown is an FSM M_X over inputs $I_2 \times U$ and outputs $V \times O_2$. Converting to the related FSM languages, we construct the associated language equation

$$L(M_A) \bullet L(M_X) \subseteq L(M_C),$$

where $L(M_A)$ is an FSM language over alphabet $I_1 \times U \times V \times O_1$, $L(M_C)$ is an FSM language over alphabet $I_1 \times I_2 \times O_1 \times O_2$ and the unknown FSM language is over alphabet $I_2 \times U \times V \times O_2$. The previous equation can be rewritten for simplicity as

$$A \bullet X \subseteq C.$$

We want to characterize the solutions of $A \bullet X \subseteq C$ that are FSM languages. It can be shown that the largest solution of the equation $A \bullet X \subseteq C$ is the language $S = \overline{A \bullet \overline{C}}$, if $S \neq \emptyset$. Unfortunately, even though A and C are FSM languages, it is not guaranteed that $S = \overline{A \bullet \overline{C}}$ is an FSM language. However, since both A and C are regular languages, S is a regular language too and a procedure can be given to obtain the largest FSM language S^{FSM} that is a solution. Therefore we have shown the largest FSM language that is a solution of the equation $A \bullet X \subseteq C$ is given by S^{FSM}, where $S = \overline{A \bullet \overline{C}}$, if $S \neq \emptyset$, and S^{FSM} is obtained from S by applying such a procedure. Finally, one can derive an FSM $M_{S^{FSM}}$ associated to S^{FSM}. This lets us talk about FSMs that are solutions of FSM equations: they are the reductions of the FSM $M_{S^{FSM}}$ (a reduction of an FSM M' is another FSM M'' whose language is equal or contained into the language of M'). So FSM M_B is a solution to the equation $M_A \bullet M_X \subseteq M_C$, where M_A and M_C are FSMs, if and only if M_B is a reduction of the FSM $M_{S^{FSM}}$ associated to S^{FSM}, where S^{FSM} is found as the largest FSM language included in the maximal solution $S = \overline{A \bullet \overline{C}}$.

Various contributions investigating partial aspects of the topic of this research have been published in the past. The authors, being from different scientific communities, noticed strong similarities between many results and therefore first tried to understand them from a single point of view. The result is this survey that attempts to unify the results and put them into a common framework and notation. The complete paper (over 30 pages) is inappropriate for publication in the workshop notes, but it will be made available on request.

Acknowledgement

The authors appreciate the support of a NATO travel grant.