Speckle Filtering of PolSAR and PolInSAR Images using Trace-based Partial Differential Equations

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Abstract— **Partial Differential Equations (PDE) filtering methods provide regularization of an image through successive iterations, where pixel values are gradually diffused in accordance with a local diffusion tensor field. The local diffusion tensor field determines the local smoothing geometry that should drive the regularization process. Most diffusion based filtering methods rely on a divergence formulation for the diffusion term which does not produce an optimal geometry preserving regularization. A new trace-based PDE approach was recently proposed and has been applied to the filtering of color images and other multivalued data. Compared to a classical divergence based formulation approach, a trace based formulation better preserves the geometric content of the image. We propose to apply this framework to the filtering of Polarimetric Interferometric Synthetic Aperture Radar (PolInSAR) images. In particular, the calculation of the local geometry is modified in order to be robust to the speckle noise. Speckle reduction performances are evaluated on both artificial and real PolInSAR images and are compared to other standard speckle reduction filters in terms of radiometric resolution improvement and meaningful details preservation.**

SAR filtering, speckle, multiscale, contourlet

I. INTRODUCTION

Speckle filtering of Polarimetric Synthetic Aperture Radar (PolSAR) images aims at removing noise while preserving the target scattering properties and meaningful high frequency features (i.e. edges, point targets, etc.). Usually, scattering properties of non deterministic targets are expressed through second-order statistics such as the covariance matrix or the coherency matrix [1]. In contrast to mono-band SAR, one hurdle to overcome with respect to PolSAR, is to properly model the speckle noise on all elements of the covariance matrix, the nature of which (multiplicative or additive) appears dependent on the correlation structure of the data [3]. Additionally, a high degree of smoothing must be achieved within extended homogeneous targets in order to derive reliable scattering property indicators [2]. Most common speckle reduction techniques are extensions of mono-channel speckle filters to the multivariate case. For example, in the enhanced Lee filter [1], elements of the covariance matrix are averaged among pixels within a sliding oriented sub-window on the condition that the window is qualified as homogeneous. Where a significant image structure is present, such as an edge between two extended targets or a point target, a weight is computed from the span image thereby enabling the preservation of deterministic polarimetric responses.

Image regularization techniques based on Partial Differential Equations (PDE), also referred to as anisotropic filtering, have been very successful in Gaussian noise removal and usually attain a high degree of smoothing which is due to their iterative nature [4]. In an anisotropic filter, the image values are iteratively diffused according to a local coefficient of diffusion which is designed to slow down the process near image discontinuities. In this paper, we explore the potential of a novel class of PDE based on a trace formulation [1]. This framework has several advantages, namely: 1) it leads to a unification of oriented gaussian filtering and PDE based regularization; 2) it provides for better control and stability of the regularization process by separating the structural information analysis from the actual regularization; and, 3) the trace-based PDE have been successfully applied to the filtering of color images where the preservation of the hue is a constraint similar to the preservation of the covariance structure of polarimetric data [7].

In our approach to explore the potential of the trace-based PDE for the filtering of PolSAR and PolInSAR images, we propose to compute a local structure tensor based on a vectorization of the Instantaneous Coefficient of Variation (ICOV) proffered by Yu [5][8]. The coefficient of diffusion, derived from the Tauber filter, is based on robust statistics [9][10]. Additionally, we compare different methods of computing the structure tensor via either the span image or by including off-diagonal terms of the covariance matrix. Finally, results are evaluated on simulated and real PolInSAR images in relation to edge preservation and smoothing performances.

II. BACKGROUND

A. Divergence based PDE

Image regularization with Partial Differential Equations is accomplished by iteratively updating the image I at each pixel position $\mathbf{x}$ according to the following evolution equation:

$$\frac{\partial I(\mathbf{x};t)}{\partial t} = \operatorname{div}\left(\mathbf{D}(\mathbf{x};t)\nabla I(\mathbf{x};t)\right) \qquad (1)$$

The local geometry of the smoothing is represented by the *local diffusion tensor* $\mathbf{D}(\mathbf{x},t)$ and acts as an "edge stopping" function meant to preserve edges. The most commonly used approaches include an isotropic diffusion function which does not contain any directional information regarding edges:

$$\mathbf{D}(\mathbf{x};t) = c(\mathbf{x};t)\mathbf{Id} \tag{2}$$

where **Id** is the identity matrix. Yu and Acton [5] proposed an anisotropic filter for SAR and ultrasound images that takes into account the multiplicative nature of the speckle noise. The proposed "edge stopping" function will slow down the diffusion process when the *Instantaneous Coefficient of Variation* (ICOV) $q(\mathbf{x};t)$ is higher than the speckle noise level $q_0(t)$

$$\begin{aligned} c(\mathbf{x};t) &= \frac{1}{1+\left[q^2(\mathbf{x};t)-q_0^2(t)\right]/\left[q_0^2(t)\left(1+q_0^2(t)\right)\right]} \\ \text{with } q^2(\mathbf{x};t) &= \frac{\left|\left\|\nabla I(\mathbf{x};t)\right\|^2/2-\frac{1}{16}\left(\Delta I(\mathbf{x};t)\right)^2\right|}{\left(I(\mathbf{x};t)+\frac{1}{4}\Delta I(\mathbf{x};t)\right)^2} \end{aligned} \tag{3}$$

The ICOV for the speckle noise ($q_0(t)$) is estimated at each iteration within a chosen homogeneous image area. In practice, several limitations of the Yu filter can be observed: 1) The convergence is possible only if the time increment is small enough ($\Delta t \leq \min\{c\}$), which is difficult to reach in case of strong speckle noise where *c* can attain very low values. Very low values for *c* will result in a very slow convergence that occasionally requires thousands of iterations; 2) when $q(\mathbf{x};t) < q_0(\mathbf{x};t)$, the coefficient of diffusion *c* is greater than one which can produce local distortions on the image; 3) The choice of a homogeneous image region in the SAR image for the computation of $q_0(\mathbf{x};t)$ can be problematic and the result may vary accordingly. Variants of the Yu filter have been suggested including the use of robust statistics in the computation of the coefficient of diffusion *c* [9][10].

B. *Trace based Regularization*

One drawback of the divergence approach (1) is that the divergence operation does not necessarily preserve the smoothing geometry initially specified by **D** [7]. To remedy this problem, Tschumperlé *et al.* [7] have proposed a unifying framework based on a very local interpretation of the regularization process [5]

$$\frac{\partial I}{\partial t} = \operatorname{trace}(\mathbf{DH}) \tag{4}$$

It has been demonstrated that this formulation is equivalent to a local convolution with an oriented gaussian:

$$I(t+dt) = I(t) * G(\mathbf{x} \mid \mathbf{D}, dt) \tag{5}$$

Where the local Gaussian *G* is oriented according to the local diffusion tensor principal directions:

$$G(\mathbf{x} \mid \mathbf{D}, dt) = \frac{1}{4\pi dt}\exp\left(-\frac{\mathbf{x}^T\mathbf{D}^{-1}\mathbf{x}}{4dt}\right) \tag{6}$$

In the following sections, we focus on the design of a local diffusion tensor that is robust to speckle noise and is capable of efficiently capturing the local geometry of PolSAR images.

III. APPLICATION TO POLSAR IMAGE FILTERING

Following, we assume that the polarimetric information is represented by a 3x3 covariance matrix (reciprocal backscattering case)

$$\begin{aligned} &\mathbf{C} = \mathbf{k}_S\mathbf{k}_S^{*T} \\ &\text{with } \mathbf{k}_S = \begin{bmatrix} S_{hh} & \sqrt{2}S_{hv} & S_{vv} \end{bmatrix}^T \end{aligned} \tag{7}$$

In the context of PolInSAR images, we have a 6x6 covariance matrix [2]:

$$\mathbf{C} = \begin{bmatrix} \mathbf{k}_{S,1} \\ \mathbf{k}_{S,2} \end{bmatrix}\begin{bmatrix} \mathbf{k}_{S,1}^{*T} & \mathbf{k}_{S,2}^{*T} \end{bmatrix} = \begin{bmatrix} \mathbf{C}^{11} & \mathbf{K}^{12} \\ \mathbf{K}^{12^{*T}} & \mathbf{C}^{22} \end{bmatrix} \tag{8}$$

For convenience, the covariance matrix is reformatted as a multivalued image **I** with the diagonal components as the first 3 channels (resp. 12 for the PolInSAR case) and using only the upper-diagonal elements (**C** is an Hermitian matrix):

$$\mathbf{I} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{22} & \mathbf{C}_{33} & \mathbf{C}_{12} & \mathbf{C}_{13} & \mathbf{C}_{23} \end{bmatrix} \tag{9}$$

A. *Polarimetric Structure Tensor*

A convenient and effective way to evaluate the structural content of a multi-valued image (dimension *d*) is to use the Di Zenzo metric [7]

$$\left\|d\mathbf{I}^2\right\| = d\mathbf{x}\mathbf{G}d\mathbf{x} \text{ with } \mathbf{G} = \sum_{n=1}^{d}\nabla I_n{}^T\nabla I_n \tag{10}$$

G is referred to as the *vector-valued structure tensor* and is the sum of the mono-channel structure tensors. The spectral decomposition of the semi-positive definite 2×2 matrix **G** provides information about the image local geometry:

$$\mathbf{G} = \lambda_+\mathbf{u}_+\mathbf{u}_+^T + \lambda_-\mathbf{u}_-\mathbf{u}_-^T \tag{11}$$

where the eigenvectors $\mathbf{u}_+$ and $\mathbf{u}_-$ respectively represent the direction of maximum change in the image (gradient) and the minimum change. The two eigenvalues λ_+ and λ_- measure the amplitude of the image fluctuations along the principal directions. If an edge is present, its strength can be measured by $\gamma = \sqrt{\lambda_+ + \lambda_-}$. In case of SAR images, the relation (10) must be modified because the gradient operator performances are strongly affected by the presence of multiplicative noise. The ICOV function (3) is more appropriate; however it is a scalar quantity unable to capture the local image geometry. Alternatively, we propose a vectorization of the ICOV (VICOV):

$$\mathbf{q}(i,j) = \left[\frac{I(i,j+1)-I(i,j-1)}{I(i,j+1)+I(i,j-1)} \quad \frac{I(i+1,j)-I(i-1,j)}{I(i+1,j)+I(i-1,j)}\right]^T \quad (12)$$

We can observe that each term is in fact the local gradient divided by the local mean which should provide some robustness to the speckle noise. This operator is also closely related to the Normalised Gradient (NG) operator proposed by Yu and Acton [8]. We define a multi-channel structure tensor as the averaging all of the mono-band structure tensors:

$$\mathbf{G}(\mathbf{x},t) = \frac{1}{d}\sum_n \mathbf{G}_n(\mathbf{x},t) = \frac{1}{d}\sum_n \mathbf{q}_n^T(\mathbf{x},t)\mathbf{q}_n(\mathbf{x},t) \quad (13)$$

$\mathbf{G}(\mathbf{x},t)$ is referred to as a tensor because the equation (13) results in a symmetric 2×2 matrix per pixel. The structure tensor is considerably noisy, therefore we can reinforce common geometric structures between the mono-band structure tensors $\mathbf{q}_n(\mathbf{x},t)$ by filtering with a 3D Gaussian kernel g_σ (the third dimension being the channel dimension):

$$\tilde{\mathbf{q}}(\mathbf{x},t) = (\mathbf{q} * g_\sigma)(\mathbf{x},t) \quad (14)$$

B. Polarimetric diffusion tensor

From the spectral decomposition of $\mathbf{G}(\mathbf{x},t)$, a diffusion tensor $\mathbf{D}$ is designed to have the same principal directions:

$$\mathbf{D} = f_+(\lambda_+,\lambda_-)\mathbf{u}_+\mathbf{u}_+^T + f_-(\lambda_+,\lambda_-)\mathbf{u}_-\mathbf{u}_-^T \quad (15)$$

The two functions f_+ and f_- must satisfy the two following constraints [7]: *1) Edge Preservation*: when $\lambda_+,\lambda_- \gg 0$ the filtering should be anisotropic and directed along the edge direction $\mathbf{u}_-$ which implies that $f_-(\lambda_+,\lambda_-) \gg f_+(\lambda_+,\lambda_-) \approx 0$ and $\mathbf{D} \simeq f_-(\lambda_+,\lambda_-)\mathbf{u}_-\mathbf{u}_-^T$; and *2) Smoothing within homogeneous areas*: when $\lambda_+ \approx 0, \lambda_- \approx 0$ the filtering should be isotropic which implies that $f_-(\lambda_+,\lambda_-) \approx f_+(\lambda_+,\lambda_-) \approx 1$ and $\mathbf{D} \simeq \mathbf{u}_+\mathbf{u}_+^T + \mathbf{u}_-\mathbf{u}_-^T$.

In the Tauber filter [9][10], a coefficient of diffusion function have been proposed based on robust statistics and on the Tukey's biweight error norm

$$f_\pm(\gamma) = \begin{cases} \left[1-\dfrac{\gamma^2}{\gamma_s^2}\right]^{2a_\pm} & \text{if } \gamma \le \gamma_s \\ 0 & otherwise \end{cases} \quad (16)$$

where γ_s is the threshold beyond which the diffusion will be halted, and is computed from the following robust statistics:

$$\begin{aligned} \overline{\gamma} &= median(\gamma) \\ \gamma_s &= \sqrt{5}\,(1.4826 \times median(|\gamma - \overline{\gamma}|) + \overline{\gamma}) \end{aligned} \quad (17)$$

The two parameters a_+ and a_- will control the degree of smoothing along the gradient direction and the edge direction respectively. Edge preservation requires that $a_+ > a_-$ so that smoothing will be less intense in the gradient direction

Once the local diffusion tensor is calculated, the resulting filtered PolSAR or PolInSAR images can be computed in two different ways [7]: 1) from the local Hessian image (4); 2) from the equivalent convolution by a local oriented Gaussian (6). In summary, an iteration of the proposed algorithm is composed of the following steps:

Step 1) Compute the VICOV values (12) on the polarimetric channels chosen for the structure detection. Three choices are possible:

a) On the span image: $Span = \sum_{n=1}^{6} I_n$;

b) On the 6 power channels;

c) On the magnitudes of all the 12 polarimetric channels composing **I**.

Step 2) In presence of a strong speckle noise, apply a 3D Gaussian filter (14) on the VICOV image to reinforce common structure.

Step 3) Compute the structure tensor (13) from the sum of all the mono-band structure tensors. For each image location, compute the structure tensor eigenvalues (λ_+,λ_-) and eigenvectors $(\mathbf{u}_+,\mathbf{u}_-)$.

Step 4) Estimate the global threshold γ_s from the structure tensor energy (17). Next, derive the diffusion functions (16) and compute the diffusion tensor (15).

Step 5) Compute the Gaussian mask (6) for a fixed size $[-M,+M]\times[-M,+M]$ at each pixel location and apply the local convolution (5) on each channel of **I**. In the event that the determinant of **D** is null, skip the convolution in respect to that particular location.

IV. RESULTS

A. Artificial PolSAR image simulation

We generate a simple simulated PolInSAR image (single look complex) using the method based on the Cholesky decomposition of the covariance matrix [2]. Five areas with different scattering properties are selected within an E-SAR image (L-band, Oberpfaffenhofen, Germany, 2 looks) for different types of targets on which we estimated the average covariance matrix. The simulated PolSAR image ($\mathbf{C}^{11}$) is shown on Fig. 1 as well as the ground truth image. The filter parameters used on the simulated image are the followings: $a_+ = 1.25$, $a_- = 0.75$, $\Delta t = 0.1$, $M = 3$ and $\sigma = 0.5$. Smoother results are obtained when the power images and the magnitudes of the 12 polarimetric channels composing **I** are employed for the structure tensor calculation instead of the span. The Yu filter was applied with $\Delta t = 0.05$ and about 2,000 iterations. Compared to the Yu filter, the proposed approach better preserves point targets. On Fig. 2, we produced results on a section of the E-SAR image ($\mathbf{C}^{11}$). Notice the distortions (circled), resulting from numerical instabilities, on the coherency map for the Yu filter result.

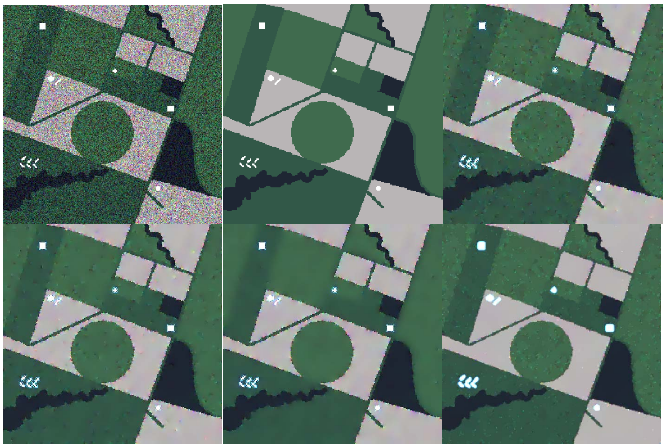

Figure 1. From left to right and top to bottom: simulated PolInSAR image ($R = |S_{hv}|, G = |S_{vv}|, B = |S_{hh}|$), Ground Truth, Trace-based (span), Trace-based (power bands), Trace based (all term magnitudes), Yu filter.

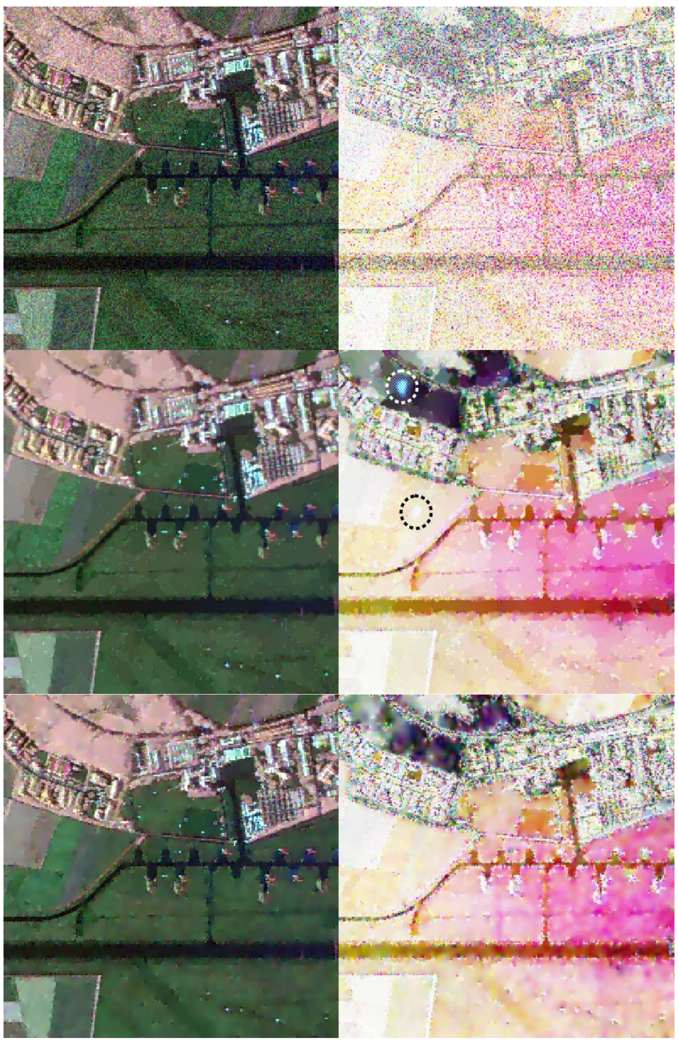

Figure 2. Filtering results on the E-SAR: The left column shows the master image ($\mathbf{C}^{11}$) and the left column is a color composite of the InSAR coherency maps ($R = \mathbf{K}^{12}_{hv}, G = \mathbf{K}^{12}_{vv}, B = \mathbf{K}^{12}_{hh}$). From top to bottom: Original Image; Yu Filter (1,500 iterations) and proposed method with the 6 power bands (465 iterations, $a_+ = 2, a_- = 1.5$, $\Delta t = 0.05$, *M=2*).

V. DISCUSSION AND CONCLUSION

We have proposed a novel approach for PolInSAR and PolSAR image filtering based on a Partial Differential Equation which is equivalent to a local filtering by an oriented Gaussian kernel. The smoothing process is driven by the analysis of the local structure of the image derived from a normalised gradient. The coefficient of diffusion is the same as the Tauber filter and is based on robust statistics of the local structure tensor energy. Additionally, this flexible framework permits the inclusion of off diagonal terms of the covariance or coherency matrix which provides a stronger smoothing result compared to the filtering based on the span image alone. The trace based approach is more stable numerically and converges faster despite longer computation time for each iteration. Compared to the Yu filter, the trace-based approach offers additional parameters which permit more efficient control of the filtering process. This preliminary evaluation of PDE based filters on PolSAR/PolInSAR images suggests promising results where a more suitable and effective compromise between a high degree of smoothing and detail preservation is reached. Future work will focus on the design of a coefficient of diffusion function more specific to polarimetric data.

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