SAR Image Filtering based on the Stationary Contourlet Transform

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Abstract— **The objective of this paper is to asess the potential of the Stationary Contourlet Transform (SCT) in relation to the issue of speckle removal in SAR intensity images. The contourlet transform can be seen as a filter bank implementation of the curvelet transform. This novel novel approach to non linear approximation aims at providing a better representation of the geometrical content of natural images. Recently, a stationary version has been proposed that preserves translation invariance. We compare the SCT performances against the curvelet transform and the stationary wavelet transform for two different speckle reduction techniques. Results indicate a better compromise between noise removal and detail preservation.**

SAR filtering, speckle, multiscale, contourlet, curvelet, wavelet

I. INTRODUCTION

Synthetic Aperture Radar (SAR) images are strongly affected by a multiplicative noise, referred to as speckle noise, which reduces the effectiveness of visual interpretation and information extraction tasks (e.g. classification). Speckle filtering traditionally aims at reaching the best compromise between noise removal and detail preservation (edges, point targets, etc.). Many wavelet-based filtering methods have been proposed in order to better separate noise from the signal (e.g. [7]). However, the wavelet transform of an image has been shown to be a sub-optimal representation for natural images where discontinuities (edges) are distributed along smooth geometric curves. Originally, the wavelet was designed as an efficient representation of 1-D discontinuities. The extension to 2-D signals was simply performed by a tensor product of two separable 1D wavelet filters.

In recent years, many new non-separable multiscale methods have been proposed in order to better capture the geometrical content of images (for example, the ridgelet and curvelet transform [1]). These transformations generate sparser and more efficient representations because they are sensitive to the directional and anisotropic content of images. Applications to speckle reduction in the curvelet domain have also been proposed [3][4][5]. However, there remain some implementation issues in respect to the digital curvelet/ridgelet transform, more particularly surrounding the signal reconstruction and the estimation of the noise level on the ridgelet/curvelet coefficients which require a Monte-Carlo estimation [1]. The *Stationary Contourlet Transform* (SCT) [6] is a shift invariant version of the critically sampled *Contourlet Transform* (CT) that has been originally proposed by Do and Vetterli [1]. The contourlet transform is a pyramidal directional filter bank that is constructed by combining the *Laplacian Pyramid* (LP) and *Directional Filter Banks* (DFB). The SCT generates an increasing number of directional high frequency images at each finer scale 2^j of the pyramid in order to satisfy the key anisotropy scaling relation for curves ($width \propto length^2$).

The objective of this paper is to asess the potential of the SCT in relation to the problem of speckle removal in SAR intensity images. This paper is organized as follows. In section II, we provide some background on the contourlet transform. In section III, we show that the contourlet coefficient probability distribution function is nearly Gaussian and we expose two established speckle reduction techniques: a Bayesian shrinkage function (MMSE) and a soft-thresholding of the coefficients of the transformation of logarithm of the SAR image. Finally, in section IV, we compare the filtering results based on the *Stationary Wavelet Transform* (SWT), the stationary contourlet transform (SCT) and the curvelet transform on an artificial image and a real SAR image.

II. THE STATIONARY CONTOURLET TRANSFORM

The *Contourlet Transform* (CT) has been proposed by Vetterli and Do and is also referred to as *Pyramidal Directional Filter Bank* (PDFB) [1]. The primary motivation is to build an efficient filter bank implementation of the curvelet transform. This consists in combining a *Laplacian Pyramid* (LP) and a *Directional Filter Bank* (DFB). The basic idea is to create a multidirection bandpass partition of the high frequency domain as shown on Fig. 1. The application of directional filters on the bandpass images produced by the LP enables the capture of directional information on discontinuities.

The original contourlet transform produces decimated high frequency images sampled at the Nyquist rate. This critical sampling is mainly required for compression applications where redundancy in the representation must be eliminated. For denoising applications, a redundant and shift-invariant will produce better results. Therefore, a non-subsampled version of the CT has been recently proposed where decimations are suppressed and filters are upsampled instead [6]. Some instances of contourlets are shown on Fig. 1 and we can observe that they offer a highly directional, localized and multiscale basis similar to curvelets.

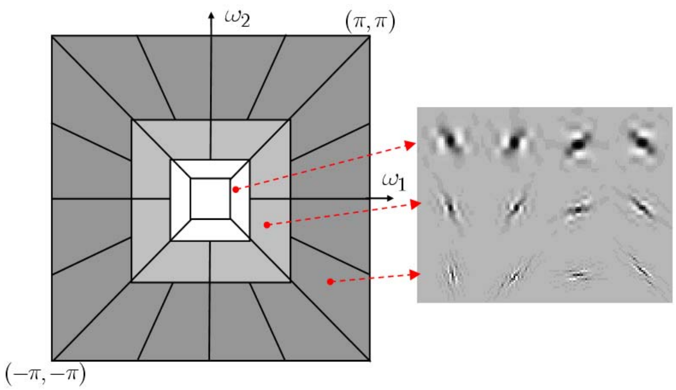

Figure 1. Frequential partitioning achieved by the contourlet transform and a few SCT basis for three different scales (top to bottom) and four different orientations (left to right).

The SCT produces 2^{l_j} high frequency images per level j where l_j is the number of levels in the DFB. Do and Veterlli [2] proposed to double the number of orientations at each finer scale. For instance, $l_1 = 4, l_2 = 3, l_3 = 2$ will produce 16 high frequency images on the finer level and 4 on the coarser level. In the following sections, we note $\left\{W_I^{[j,k]}\right\}_{k=1,\ldots,2^{l_j}}$ the set of contourlet coefficients for the level j.

III. Application to SAR Image Filtering

A. *Influence of the Speckle Noise on the Contourlet Coefficients*

We briefly recall some statistical characteristics of SAR images pertinent to our analysis. Speckle noise is a natural consequence of the coherent nature of SAR illumination. Interferences of the backscattered signals from elementary scatterers composing the resolution cell produce strong random intensity fluctuations. A good model for the observed intensity is the multiplicative model where the random process describing the observed intensity I is the product of two independent random variables:

$$I = SR \tag{1}$$

where R and S are the random variables describing respectively the radar reflectivity and speckle noise produced by the coherent imaging. Both random variables are assumed to be gamma distributed: $R \overset{d}{\sim} \Gamma(\nu, \nu/\mu_R)$, $S \overset{d}{\sim} \Gamma(L, L)$ where ν, μ_R and L are the texture parameter, the radar mean reflectivity and the number of looks respectively. Like most speckle filters, we assume white random processes (no spatial correlation) which

$$\mathrm{var}_I = \mu_I^2(1 + C_S)C_R + \mu_I^2 C_S \tag{2}$$

where $C_S = 1/L$ and $C_R = 1/\nu$ are the squared coefficients of variation of the speckle and the radar reflectivity respectively. One convenient way to deal with the multiplicative model is to define a pseudo additive noise N:

$$\begin{aligned} I &= R + N \\ N &= R(S-1) \end{aligned} \tag{3}$$

The noise N is signal dependent. The application of the SCT on I can then be split in two contributions:

$$W_I^{[j,k]} = W_R^{[j,k]} + W_N^{[j,k]} \tag{4}$$

In the following, we note $\mathrm{var}_I^{[j,k]}$ the variance of $W_I^{[j,k]}$. In [7], relationships between moments of the high frequency images and moments of the original image are expressed as:

$$\begin{aligned} \mathrm{var}_R^{[j,k]} &= S_2^{[j,k]}\mu_R^2 C_R \\ \mathrm{var}_N^{[j,k]} &= S_2^{[j,k]}\mu_R^2 C_S(1 + C_R) \end{aligned} \tag{5}$$

where $S_2^{[j,k]}$ is computed from the equivalent high frequency filter response of the contourlet (or wavelet) transform:

$$W_I^{[j,k]} \simeq I * \psi^{[j,k]} \Rightarrow S_2^{[j,k]} = \sum_i \left(\psi_i^{[j,k]}\right)^2 \tag{6}$$

In order to determine a proper statistical model for describing the contourlet coefficients we simulate a speckled image with a constant reflectivity ($C_R = 0$). The observed distributions of contourlet coefficients are shown on Fig. 2 (only for one direction). The Gaussian model is centered (zero mean) and its variance is derived from (5):

$$\mathrm{var}_N^{[j,k]} = S_2^{[j,k]}\mu_R^2 C_S \tag{7}$$

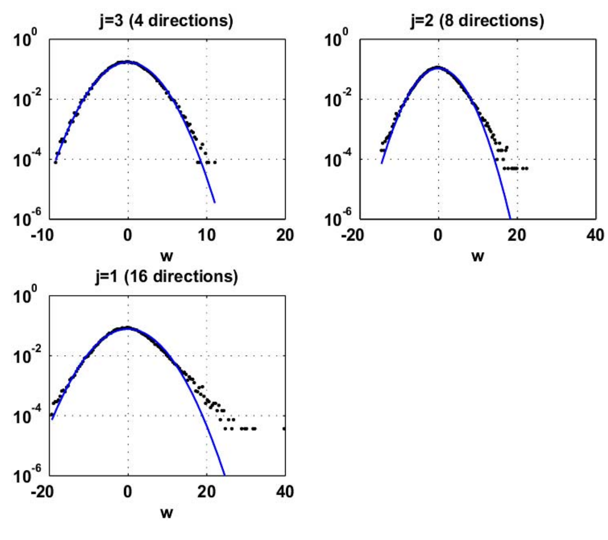

Figure 2. Observed histograms on one contourlet coefficient image $W_I^{[j,1]}$ and 3 successive scales for a simulated uniform speckle image (*L=1*). The blue lines represent the gaussian pdf. A logarithmic scale is used for the probability value in order to enhance tail behaviors.

As seen on Fig. 2, the observed contourlet probability distribution functions at different scales are in congruous agreement with the Gaussian model.

B. *Weighting of the Contourlet Coefficients*

1) Bayesian shrinkage: The weighting factor will shrink noisy contourlet coefficients:

$$\hat{W}_R^{[j,k]}(x,y) = g^{[j,k]}(x,y) \times W_I^{[j,k]}(x,y) \tag{8}$$

In [7], we applied a MMSE criterion equivalent to a Maximum *a posteriori* criterion (MAP) with Gaussian distributions. The resulting solution is a Wiener-like weighting function:

$$g^{[j,k]} = \max\{0, \frac{\mathrm{var}_R^{[j,k]}}{\mathrm{var}_R^{[j,k]} + \mathrm{var}_N^{[j,k]}}\} \tag{9}$$

In order to better preserve strong edges and point targets, we preserve high energy wavelet/contourlet:

$$g_k^{[j]} = \begin{cases} \max\{0, \dfrac{\hat{C}_I^{[j,k]} - 1}{\hat{C}_I^{[j,k]}(1+C_S)}\} & ,\hat{C}_I^{[j,k]} < 2+L \\ 1 & ,\hat{C}_I^{[j,k]} \geq 2+L \end{cases} \tag{10}$$

Where $\hat{C}_I^{[j,k]}$ is the coefficient of variation of the high frequency images estimated within a local sliding window [7]:

$$\hat{C}_I^{[j,k]} \stackrel{\Delta}{=} \frac{\widehat{\mathrm{var}}_I^{[j,k]}}{S_2^{[j,k]} \mu_I^2 C_S} \tag{11}$$

2) Soft thresholding: For a logarithmically transform SAR image intensity, the speckle noise becomes additive:

$$\tilde{I} = \ln(I) = \tilde{S} + \tilde{R} \tag{12}$$

The variance and mean of $\tilde{S}$ has been derived by Xie et al. [8] for an integer number of looks:

$$\mu_{\tilde{S}}^2 = -0.577215 - \ln(L) + \sum_{m=1}^{L-1} \frac{1}{m}$$

$$\mathrm{var}_{\tilde{S}} = \frac{\pi^2}{6} - \sum_{m=1}^{L-1} \frac{1}{m^2}$$

For speckle reduction on the logarithmic transformation of *I*, we apply a soft thresholding technique on the wavelet/contourlet coefficients of $\tilde{I}$ as follows:

$$\hat{W}_{\tilde{R}}^{[j,k]} = \mathrm{sgn}(\hat{W}_{\tilde{I}}^{[j,k]}) \max\left(0, \left|\hat{W}_{\tilde{I}}^{[j,k]}\right| - T^{[j,k]}\right) \tag{13}$$

We choose a threshold function of the original noise variance of $\tilde{S}$: $T^{[j,k]} = S_2^{[j,k]} \mathrm{var}_{\tilde{S}}$.

IV. RESULTS

A. *Simulated Image*

The artificial SAR image is simply formed by adding multiplicative noise on the *barbara* image shown on Fig. 3. We chose this image because of its important high frequency content. The implementation of the Curvelet transform is described in [1]. We use biorthogonal filters (5/3) and three levels of decomposition for all the transformations. The initial tile size is 16x16 for the curvelet transform. The contourlet transform produces 16, 8 and 4 high frequency images for the levels 1 to 3 respectively ($l_1 = 4, l_2 = 3, l_3 = 2$). The MMSE shrinkage factor (10) is applied on the contourlet (SCT-MMSE) and wavelet coefficients (SWT-MMSE) only. The local statistics are estimated within a local window of size $D^{[j]} \times D^{[j]}$ with $D^{[j]} = 2^{j-1} + 1$. The soft thresholding is applied in the contourlet (SCT-Soft), wavelet (SWT-Soft) and curvelet domain (Curv-Soft). The restoration performances are evaluated in terms of PNSR improvements (see Table 1). We can observe that the SCT attains higher PSNR, in particular when the soft thresholding is used. The SCT-Soft is applied with two fitlers: a short biorthogoanl filter (5/3) and a larger one (pkva6 [9]).

TABLE I. FILTERING PERFORMANCES GIVEN IN $\Delta PSNR = PSNR_{\hat{I}} - PSNR_I$ (DB)

	Original	MMSE		Soft-thresholding		
L	$PSNR_I$	***SWT***	***SCT***	***SWT***	***SCT***[1]	***Curv***
1	5.96	9.85	8.94	9.08	10.82 (10.06)	7.37
2	8.89	9.14	8.93	9.23	9.95 (9.79)	7.93
3	10.67	8.48	8.70	8.78	9.24 (9.38)	7.87
4	11.91	7.80	8.10	8.55	8.71 (9.02)	7.93
8	14.93	6.77	7.11	7.52	7.26 (7.95)	7.55

Figure 3. Original image (left) and speckle corrupted image, *L=8* (right).

Figure 4. from left to right and top to bottom: SWT-MMSE, SCT-MMSE, SWT-Soft, SCT-Soft (5/3), SCT-soft (pkva filters), and Curv-Soft.

[1]Numbers between parenthesis are for the SCT using the pkva6 filter.

B. Real image

The filters are applied on a 256x256 E-SAR image of Oberpfaffenhofen, Germany (L-band, 1.5mx1.5m) provided by the DLR (see Fig. 5). The image has been resampled in azimuth and decimated by a factor 2 (the resampling process leads to an effective number of looks equals to 2 looks). After filtering, the equivalent of looks is estimated within the area indicated by the white rectangle on Fig. 5. When the MMSE filter is applied, the SWT produces a superior visual result compared with the SCT, presumably because the SCT basis has a larger spatial support. However, when the soft thresholding is used, the SCT produces a smoother result. The estimated ENL are the following: SWT-MMSE (27), SCT-MMSE (13), SWT-Soft (63), SCT-Soft (86), Curv-Soft (22).

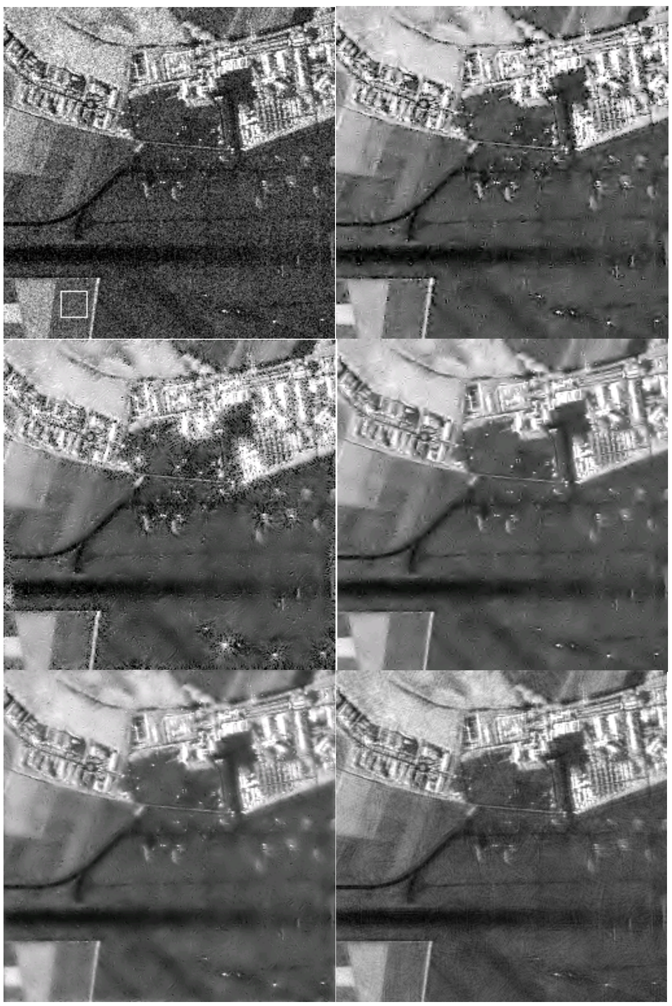

Figure 5. Results on the E-SAR image (HH), from left to right and top to bottom (in dB): original, SWT-MMSE, SCT-MMSE, SWT-Soft, SCT-Soft and Curv-Soft

V. CONCLUSIONS

In this paper, we assessed the potential of the stationary contourlet transform to remove speckle noise in SAR images. The filtering was performed by applying two different techniques: a MMSE shrinkage function based on local statistics and a soft-thresolding on the contourlet coefficients of the homomorphic transform. The SCT appears to obtain superior results on both the artificial and the real image. When compared to the curvelet transform, we observed less artifacts within homogeneous image areas. The local MMSE criterion was less effective when used with the SCT and should only be used with short wavelets. When used with a short filter (e.g. biorthogonal 5-3), the SCT performance in restoring linear details is visually equivalent to the SW. Conversely, the use of a larger filter (e.g. pkva) produces an improved restoration of linear features but an increased number of artifacts appear within smooth image areas.

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