# SAR IMAGE FILTERING VIA LEARNED DICTIONARIES AND SPARSE REPRESENTATIONS

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**ABSTRACT**

In the last decade there has been a growing interest in the study of sparse representation of signals. In particular, many new multiscale image representations in a geometric space have been proposed (Curvelets, Ridgelets, Contourlets, etc.). Instead of using a fixed transformation, an alternative approach is to build a sparse dictionary from the signal itself. In the present work, we propose a novel approach for speckle noise reduction in SAR images using a sparse and redundant representation over trained dictionaries. In this approach, an adaptive dictionary composed of image patches (called atoms) is learned from the image so that it constitutes a sparse representation of the image content. This learning process, called K-SVD, is efficiently performed using an Orthogonal Matching Pursuit (OMP) and a Singular Value Decomposition (SVD). This new approach is effective in removing white additive Gaussian noise despite the fact that elements of the dictionary are learned from the noisy image, the algorithm is converging toward meaningful atoms that are already showing a reduction in noise level.

***Index Terms*—** K-SVD, speckle filtering, SAR

## 1. INTRODUCTION

Recent advances in speckle filtering have benefited from the use of redundant multiscale representations [1]-[4]. The Stationary Wavelet Transform (SWT) can be interpreted as a projection of the signal on an overcomplete or redundant dictionary of predefined signals called atoms. One advantage of these representations is that they produce a sparse representation of the signal where a few large coefficients are enough to capture the meaningful information. Furthermore, the constraint of sparsity and redundancy in the image domain has lead to emergence of many new multiscale representations such as Contourlets, Ridgelets, Curvelets and many others [3]. Another approach is to build a sparse dictionary, or codebook, directly from the image iteself. Recently, Aharon *et al.* [7] proposed the K-SVD algorithm as a generalization of the K-means algorithm for the iterative construction of a dictionary of proptype signal-atoms and the sparse coding of image. Applied to the denoising of Gaussian additive noise, superior results where obtained compared to fixed dictionaries [7][8][9].

In this paper, we are proposing to use a sparse and redundant dictionary in order to filter SAR images. One particular challenge is to deal with the multiplicative nature of the speckle noise. We investigate two different ways to filter the SAR images: 1) the filtering of the log transform of the SAR image and 2) the direct filtering in SAR intensity domain. Results are evaluated on both artificial and real images.

## 2. K-SVD

### 2.1 Background

More details about the K-SVD algorithm can be found in [7][8][9]. The so-called *Sparseland model*, defines a dictionary $\mathbf{D} \in \Re^{n \times K}$ composed of *K* atoms of size *n*. K-SVD is looking for the sparsest representation $\boldsymbol{\alpha}$ of a signal **y** that gives the best signal representation

$$\hat{\boldsymbol{\alpha}} = \arg\min \|\boldsymbol{\alpha}\|_0 \text{, subject to } \|\mathbf{y}\text{-}\mathbf{D}\boldsymbol{\alpha}\|_2 \leq \varepsilon \quad (1)$$

where $\|.\|_0$ counts non zero coefficients. The algorithm alternates a sparse coding step based on an *Orthogonal Matching Pursuit* (OMP) and a dictionary updating step based on a simple *Singular Value Decomposition* (SVD). In particular, if *K=1* we get the well known K-means algorithm. Two interesting properties relevant for speckle reduction: 1) the OMP step rejects image patches when the approximation is below the noise level $\varepsilon$ therefore reducing the impact of noise; 2) the SVD step will combine patches in order to construct better atoms leading to further noise reduction.

### 2.2 Application to image filtering

The K-SVD has been applied with great success to the denoising of images corrupted by additive white Gaussian noise (AWGN) [8][9][11]. A generalization of a MAP estimator can be proposed [11]

$$\{\hat{\alpha}_{ij}, \hat{\mathbf{D}}, \hat{\mathbf{x}}\} = \underset{\mathbf{D},\mathbf{x},\alpha_{ij}}{\arg} \min \lambda \|\mathbf{x} - \mathbf{y}\|_2^2 + \sum_{i,j} \mu_{ij} \|\alpha_{ij}\|_0 + \sum_{i,j} \|\mathbf{D}\alpha_{ij} - \mathbf{R}_{ij}\mathbf{x}\|_2^2 \quad (2)$$

A simple analytic solution can be derived

$$\hat{\mathbf{X}} = \left(\lambda \mathbf{Id} + \sum_{i,j} \mathbf{R}_{ij}{}^T \mathbf{R}_{ij}\right)^{-1} \left(\lambda \mathbf{Y} + \sum_{i,j} \mathbf{R}_{ij}{}^T \hat{\mathbf{D}} \hat{\boldsymbol{\alpha}}_{ij}\right) \quad (3)$$

where $\mathbf{R}_{ij}$ if the operator that puts back the patch at its position *i,j* in the image and **Id** is the identity operator. The filtering method consists in two steps: 1) learn the dictionary directly from the noisy image patches using K-SVD; 2) compute the sparse image representation on **D** (equation (1)) and the filtered image using (3).

## 3. FILTERING OF SAR LOG-INTENSITY IMAGES

We briefly recall some statistical characteristics of SAR images pertinent to our analysis. Speckle noise is a natural consequence of the coherent nature of SAR illumination. Interferences of the backscattered signals from elementary scatterers composing the resolution cell produce strong random intensity fluctuations. A good model for the observed intensity is the multiplicative model where the random process describing the observed intensity *I* is the product of two independent random variables:

$$I = SX \quad (4)$$

where $X$ and $S$ are the random variables describing respectively the radar reflectivity and speckle noise produced by the coherent imaging. Both random variables are generally assumed to be gamma distributed: $X \overset{d}{\sim} \Gamma(\nu, \nu / \mu_X)$, $S \overset{d}{\sim} \Gamma(L, L)$ where $\nu$, $\mu_X$ and $L$ are the texture parameter, the radar mean reflectivity and the number of looks respectively. Like most speckle filters, we assume white random processes (no spatial correlation) which

$$\mathrm{var}_I = \mu_I^2 (1 + C_S) C_X + \mu_I^2 C_S \quad (5)$$

Where $C_S = 1/L$ and $C_X = 1/\nu$ are the squared coefficients of variation of the speckle and the radar reflectivity respectively. One convenient way to deal with the multiplicative model is to define a pseudo additive noise *N*:

$$\begin{aligned} I &= X + N \\ N &= X(S - 1) \end{aligned} \quad (6)$$

The noise *N* is signal dependent. For a logarithmically transformed SAR image intensity, the speckle noise becomes additive:

$$\tilde{I} = \ln(I) = \tilde{S} + \tilde{X} \quad (7)$$

The variance and mean of $\tilde{S}$ has been derived by Xie *et al.* [8] for an integer number of looks:

$$\begin{aligned} \mu_{\tilde{S}}^2 &= -0.577215 - \ln(L) + \sum_{m=1}^{L-1} \frac{1}{m} \\ \mathrm{var}_{\tilde{S}} &= \frac{\pi^2}{6} - \sum_{m=1}^{L-1} \frac{1}{m^2} \end{aligned} \quad (8)$$

Note that the filtering (3) is not satisfying for SAR images because it introduces a potential bias if $\lambda > 0$ ($E[\hat{\mathbf{X}}] \approx \frac{n}{\lambda + n} E[\mathbf{X}]$) in addition to the bias (8) introduced by the logarithmic transformation. In practice, the K-SVD is not applied on the patch directly but on the mean centered patch, so the filtering equation is in fact the following

$$\hat{\hat{\mathbf{X}}} = \left(\sum_{i,j} \mathbf{R}_{ij}{}^T \mathbf{R}_{ij}\right)^{-1} \left(\sum_{i,j} \mathbf{R}_{ij}{}^T (\hat{\mathbf{D}} \hat{\boldsymbol{\alpha}}_{ij} + <\tilde{\mathbf{I}}>_{ij})\right) \quad (9)$$

The effect of the operator $\left(\sum_{i,j} \mathbf{R}_{ij}{}^T \mathbf{R}_{ij}\right)^{-1} \sum_{i,j} \mathbf{R}_{ij}{}^T$ will be to average contributions from overlapping patches for a given pixel position resulting in further image smoothing. The first term is a local averaging of reconstructed zero-centered image patches $\hat{\mathbf{D}} \hat{\boldsymbol{\alpha}}_{ij}$ overlapping the same pixel position and $<\tilde{\mathbf{I}}>$ is the local mean. The above equation is very similar to a local MMSE filter [10]

$$\hat{\hat{\mathbf{X}}} = \max\left\{\frac{\sigma_{\tilde{\mathbf{I}}}{}^2 - \sigma_{\tilde{\mathbf{S}}}{}^2}{\sigma_{\tilde{\mathbf{I}}}{}^2}, 0\right\} (\tilde{\mathbf{I}} - <\tilde{\mathbf{I}}>) + <\tilde{\mathbf{I}}> \quad (10)$$

except that the high frequency component is now derived from the K-SVD algorithm.

## 4. FILTERING OF SAR INTENSITY IMAGES

The direct filtering of SAR images in intensity format is more difficult due to the non stationarity of the signal-dependent noise. In case of non homogeneous noise, a solution was proposed by Mairal *et al.* [6] where the local variance is stabilized. The OMP step has to be modified

$$\hat{\boldsymbol{\alpha}} = \arg\min \|\boldsymbol{\alpha}\|_0, \quad \text{subject to } \|\beta \otimes (\mathbf{y}\text{-}\mathbf{D}\boldsymbol{\alpha})\|_2^2 \leq \varepsilon^2 \quad (11)$$

where $\beta$ is a variance stabilizing factor and $\otimes$ the element-wise multiplication of two vectors. From the multiplicative model for SAR images, we can define $\beta$ has the inverse of expected noise level assuming an homogeneous patch with fully developed speckle

$$\hat{\beta}_{ij} = \frac{\sqrt{L}}{<I>_{ij}} \quad (12)$$

# 5. RESULTS

## 5.1. Artificial images

We evaluate the two proposed filtering methods: K-SVD on the logarithm of the SAR image and on the variance-stabilized intensity image. Simulated speckled image are generated with various level of speckle noise as shown on Fig. 1.

An important parameter is the threshold $\varepsilon$ in (1) and (11) which defines the quality of the approximation by K-SVD. If $\varepsilon$ value is too high, then few patches will be retained for the construction of **D**. Conversely, if $\varepsilon$ is too low then too many noisy patches containing no significant details will be considered. The optimal choice for $\varepsilon$ will depend on the atom size *n* and the noise distribution. We estimate $\varepsilon$ based on a Monte-Carlo procedure where noisy patches **w** are simulated and a confidence level of 93%:

$$P(\|\mathbf{w}\|_2^2 \leq \varepsilon^2) = 0.93 \qquad (13)$$

For the direct filtering of the intensity images (section 4), we get a value for $\varepsilon$ around *1.22* for *n=25* which is independent of the number of looks. In all the experiments, the dictionary is composed of 100 5x5 atoms and was initialized by taking random patches within the noisy image. The learning converges usually after 5 iterations.

Figure 1. Original image (left) and speckle corrupted image, *L=8* (right).

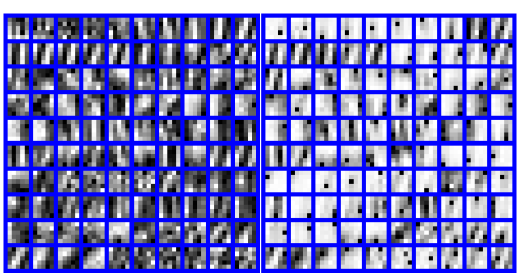

Figure 2. Dictionaries learned on the noisy image shown on Fig. 1: on the log of the intensity (left) and on the variance stabilized intensity image (right).

Filtering performances in terms of Peak-SNR (Table I) are compared with methods based on soft thresholding of fixed dictionary projections: Stationary Wavelet (SWT), Curvelets (Curv) and Stationary Contourlet (SCT) transforms (see reference [3] for details).

TABLE I. FILTERING PERFORMANCES GIVEN IN $\Delta PSNR = PSNR_{\hat{I}} - PSNR_I$ (DB)

| | K-SVD | | Soft-thresholding | | |
|---|---|---|---|---|---|
| ***L*** | ***Log-I*** | ***I*** | ***SWT*** | ***SCT*** | ***Curv*** |
| 1 | 12.28 | 9.48 | 9.08 | 10.82 | 7.37 |
| 2 | 11.08 | 8.96 | 9.23 | 9.95 | 7.93 |
| 3 | 10.15 | 8.65 | 8.78 | 9.24 | 7.87 |
| 4 | 9.74 | 8.01 | 8.55 | 8.71 | 7.93 |
| 8 | 8.93 | 7.66 | 7.52 | 7.26 | 7.55 |

Figure 3. from left to right and top to bottom: SWT-MMSE, SWT-Soft, SCT-soft, Curv-Soft and proposed methods (K-SVD on log-I and I).

A higher degree of smoothing is achieved by K-SVD applied on the log-intensity compared to the soft thresholding of fixed dictionary transforms. In addition, less artifacts are observed within homogeneous areas. When learning is performed on the intensity image, the resulting atoms are clearly more influenced by noise despite the variance stabilization as we can see on Fig. 2.

## 5.2. Real images

The filters are applied on a 256x256 E-SAR image of Oberpfaffenhofen, Germany (L-band, 1.5mx1.5m) provided by the DLR (see Fig. 4). The image has been resampled in azimuth and decimated by a factor 2 (the resampling process leads to an effective number of looks equals to 2 looks). After filtering, the equivalent of looks is estimated within the area indicated by the white rectangle on Fig. 4. We give also filtering results obtained using fixed dictionaries and soft thresholding [3]. The estimated ENL are the following: SWT-Soft (63), SCT-Soft (86), Curv-Soft (22), K-SVD on log-I (95), K-SVD on the intensity image (36).

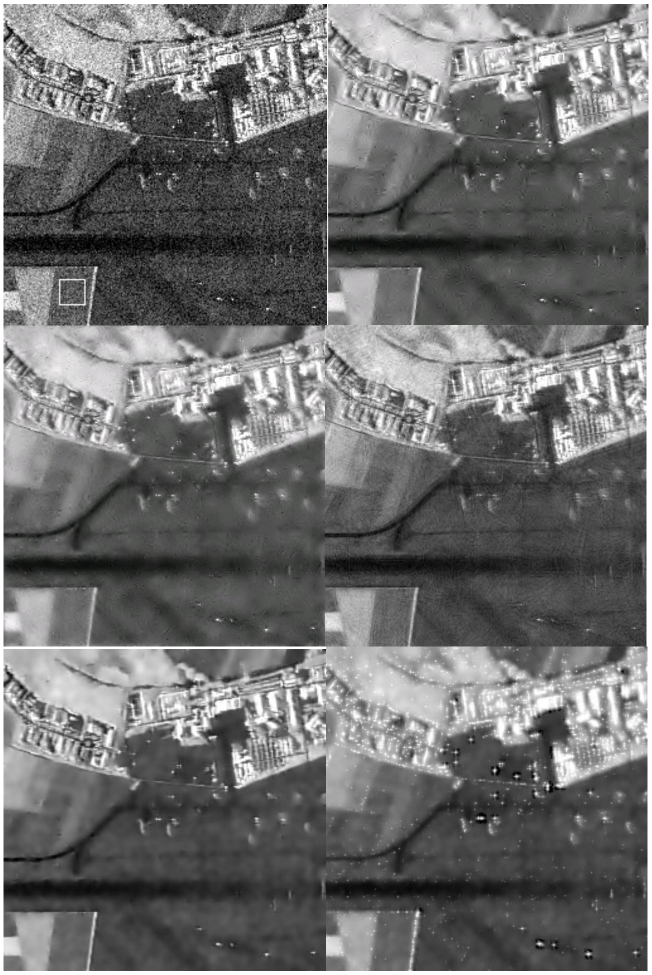

Figure 4. Results on the E-SAR image (HH), from left to right and top to bottom: original, soft thresholding of the Stationary Wavelet Transform (SWT), Stationary Contourlet Transform (SCT) and Curvelet transform (Curv). The last row shows the filtering results based on K-SVD on the log-intensity (left) and intensity (right).

## 6. CONCLUSIONS

We investigated the use of K-SVD for the removal of speckle noise on SAR images in intensity and log-intensity format. K-SVD derives an overcomplete dictionary directly from the noisy image and offers a sparse representation of the image content.

Better filtering results were produced when filtering in the log domain. The quality of detail preservation is close to what can be obtained using fixed overcomplete dictionaries (curvelets, contourlets, wavelets). In addition, less artifacts in homogeneous image regions are present compared to Curvelets based filtering for instance.

The learning of the dictionary directly from the variance stabilized SAR intensity image is more difficult and the resulting atoms are visibly influenced by the speckle noise especially for a low number of looks (see Fig. 2). In addition, artifacts around point targets were observed on the E-SAR image.

Also, the impact of the value of the various K-SVD parameters (atoms and dictionary size) should be investigated.

Many improvements are possible; in particular it could be interesting to learn a multiscale dictionary using different patch sizes instead on one fixed size [11]. Also, we are currently exploring the possibility of using K-SVD for the filtering of Polarimetric SAR images.

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